

Bridge damage classification using multiple responses of vehicles and 1-D convolutional neural networks

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ABSTRACT: Bridges exposed to extreme environmental conditions are susceptible to damage and even failure during service life. Traditional monitoring techniques may necessitate the installation of numerous sensors on the bridge, which can be time-consuming and costly. Instead, the indirect method typically employs several accelerometers attached to the passing vehicle, which is more economical and more accessible to operate. To promote the development of the indirect method, this paper proposes a novel vehicle vibration-based method for classifying bridge damage of varying severity using cutting-edge deep learning techniques. Initially, the framework for damage classification based on the responses of a single vehicle and 1-dimensional convolutional neural networks (1-D CNNs) is appropriately designed and introduced. Then, the proposed approach is evaluated using a steel continuous beam and a model truck in the laboratory, which is utilized to simulate a vehicle-bridge interaction (VBI) system in engineering applications. The experimental results indicate that the bridge's damage severity can be predicted by the CNN with high accuracy, thereby validating the inclusion of bridge damage information in the passing vehicle's responses. Furthermore, it is determined that employing multiple responses from the vehicle facilitates the improvement of damage classification accuracy. Heavier vehicles are conducive to the transfer of more bridge-damaged information and are therefore recommended in engineering.

1 INTRODUCTION

Bridge structures play a vital role in European transportation systems, yet their aging and deterioration pose significant challenges. Over the past decades, the need for structural health monitoring (SHM) of bridges has become increasingly apparent. Traditional monitoring heavily relies on visual inspections by experienced engineers, but this approach becomes impractical for newly constructed bridges spanning tens or even hundreds of meters. In the 21st century, a promising and expedient solution to bridge health monitoring has emerged through the vibration-based approach (Hou and Xia, 2021). This method involves monitoring the modal parameters throughout the bridge's service life, offering insights into potential damage occurrences. However, the conventional approach mandates the installation of a large number of sensors, including temperature sensors, accelerometers, strain gauges, etc. directly on the bridge (referred to as the direct method) to establish a sensing network. Undoubtedly, this solution incurs a high cost and is typically reserved for significant projects rather than short- and mid-span bridges.

Over the past two decades, researchers worldwide have developed the indirect method, involving the installation of sensors on passing vehicles rather than directly on bridges (Malakjafarian et al., 2022; Wang et al., 2022; Xu et al., 2024; Yang et al., 2024). This approach is characterized for its economic efficiency and convenient applications. Numerous numerical simulations and experiments have demonstrated the viability of extracting bridge modal parameters from vehicle responses (Feng et al., 2023; Li et al., 2023a). However, practical

engineering observations reveal that modal parameters of bridges may not be highly sensitive to damage due to the influence of operational conditions, such as temperature effects and ongoing traffic. Consequently, bridge health monitoring based on changes in modal parameters may face essential challenges.

In recent years, artificial intelligence (AI) has made significant strides across various domains, notably enhancing bridge health monitoring. Traditional direct methods initially employed intelligent techniques such as supervised deep learning (DL) and transfer learning (TL), yielding robust results in bridge health monitoring. When applying the indirect method, vehicle responses typically include vehicular information, road roughness, and bridge vibrations, with the latter being particularly challenging to identify (González et al., 2023). DL techniques are good at extracting essential features, making them valuable for identifying bridge information from passing vehicle responses. Malekjafarian et al. (2019) developed a machine learning approach using vehicle accelerations to detect and quantify bridge damage. Two strategies, involving time-domain or frequency-domain responses from multiple vehicle runs, successfully identified damage occurrences and provided references for damage severity. Corbally and Malekjafarian (2022) improved this method by incorporating contact-point (CP) frequency responses into artificial neural networks (ANNs), demonstrating superior performance in identifying bridge damage. The proposed method underwent robust testing with variations in vehicle speeds, ambient temperatures, and road roughness levels. In 2023, Li et al. (2023b) emphasized the importance of considering both low and high-frequency responses of passing vehicles for extracting bridge damage-sensitive features. Furthermore, their findings verified that short-time vibrations during vehicular passage contain valuable information for determining the bridge's health state (Li et al., 2023c). To overcome the challenges that DL techniques typically require a large number of samples, a physics-guided framework was proposed by Lan et al. The results verified the effectiveness of the proposed method in damage indication, quantification, and localization (Lan et al., 2024). Despite these advancements, most studies have simplified vehicles into a quarter-car model during training (Corbally and Malekjafarian, 2023). The vehicle's response along with one degree of freedom (DOF) is considered. In practice, vehicles have multiple DOFs, and each can contain bridge-related information (Li et al., 2023d). Therefore, for improved accuracy in the training process, responses from multiple positions of the vehicle need to be considered.

This paper introduces an innovative approach to damage classification using 1-dimensional convolutional neural networks (1-D CNNs) to advance the monitoring process. Sensors are strategically positioned on various locations of a passing vehicle, collecting its accelerations as it traverses the bridge. Subsequently, the time-domain vehicular accelerations obtained from multiple positions are transformed into the frequency domain. These frequency-domain responses are then employed as inputs across different channels of the 1-D CNN for damage severity classification. To illustrate the effectiveness of the proposed method, an experimental study involving a U-shape beam and a scaled truck is conducted. The subsequent sections of this paper are structured as follows: Section 2 delves into the theoretical foundation of employing 1-D CNNs for damage classification within the indirect method. Section 3 outlines the laboratory experimental setups for the vehicle-bridge interaction (VBI) system and introduces artificial damage cases. Results and discussions are offered in Section 4. Finally, Section 5 provides the conclusion of this paper, summarizing the key findings and potential avenues for future research.

2 THEORY FOUNDATION

2.1 *Vehicle's multiple responses*

This paper introduces a novel approach using multiple responses of the vehicle, distinguishing itself from existing studies. Assuming a vehicle equipped with N accelerometers, denoted as \ddot{z}_i for the accelerations measured by the i th sensor, this study advocates optimal sensor placement aligning with the 3-D vehicle's seven degrees of freedom, as detailed in the reference (Li

et al., 2023d). However, practical challenges arise in accurately localizing the vehicle body’s gravity center and capturing vibrations from the vehicular wheels, particularly given the intricate suspension systems. As an initial study, this paper employs a pragmatic solution, utilizing only two accelerometers strategically placed on the front and rear axles, denoted as $N = 2$.

Further, in order to remove the influence of unrelated information about the bridge, only when the vehicle is on the bridge will the vehicle’s accelerations be recorded. This can typically be achieved by installing GPS sensors on the passing vehicle (Lan et al., 2023). Signals collected by the vehicle could be truncated according to location information. Assume that the vehicle’s rear wheels enter the bridge at T_{in} and its front wheels leave the bridge at T_{out} , and only the vehicle’s accelerations within T_p will be utilized, where T_p is the vehicle’s passing time and is shown in Eq. (1),

$$T_p = T_{out} - T_{in} \quad (1)$$

2.2 1-D CNNs

In contrast to traditional 2-D CNNs tailored for image analysis, 1-D CNNs specialize in handling single-dimensional inputs. While standard images possess three channels—R, G, and B—the proposed method incorporates multiple signal channels derived from diverse positions on the moving vehicle. The architecture of 1-D CNNs encompasses convolutional layers, 1-D max-pooling/average pooling layers, and fully connected layers. The computations within the convolutional layers can be expressed by Eqs. (2) and (3) (Abdeljaber et al., 2018),

$$x_k^l = b_k^l + \sum_{i=1}^{N_{l-1}} \text{CONV}(w_{ik}^{l-1}, s_i^{l-1}) \quad (2)$$

$$y_k^l = f(x_k^l) \quad (3)$$

where x_k^l means the input of the l th layer. b_k^l represents the bias of the k th neuron at layer l . $\text{CONV}(\cdot)$ denotes the 1-D convolution without zero padding. w_{ik}^{l-1} is the 1-D kernel from the i th neuron at layer $l - 1$ to the k th neuron at layer l . f means the activation function. Typical activation functions include rectified linear unit (ReLU), Sigmoid, and Tanh.

1-D max-pooling/average-pooling layers efficiently down-sample the data by selecting the maximum/average value within a defined neighborhood. Interested readers can find further details in the reference (Murray and Perronnin, 2014). The resulting output is flattened into one dimension before being connected to fully connected layers. The ultimate output of the fully connected layer corresponds to the different damage severities in this paper.

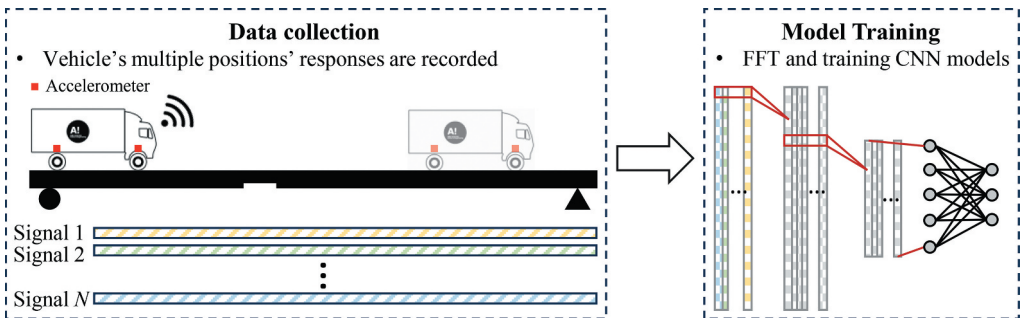


Figure 1. Data collection and model training.

3 LABORATORY EXPERIMENTS

3.1 *Vehicle models*

This paper employs a meticulously scaled model truck to simulate real-world engineering vehicles. Figures 2 and 3 illustrate the truck, its system and installed accelerometers. The employed truck model is the Tamiya Mercedes-Benz 1850L, with a scale ratio of 1:14. The truck's self-mass, denoted as T_0 , is 4.305 kg, and to simulate heavy vehicles, an additional mass of 5.157 kg is added (referred to as T_5). A guide-wire system ensures the truck follows a straight path without colliding with the beam's flange. The bottom view of the scaled truck is depicted in Figure 3, showcasing its scaled suspension system, engine, connecting shaft, and more. Driven by a 540-brushed electric motor powered by a Tamiya Ni-MH 7.2 V–3000 mAh battery, the vehicle introduces engine noise that inevitably affects acceleration data collection in engineering. Two Brüel & Kjær accelerometers (type 4371) are attached to the front and rear axles (Figure 3), with a sampling frequency set at 10 kHz. The vehicle, remotely controlled, exhibits slight speed variations across runs due to battery constraints. However, to allow for optimal vibrations, the wire tension is deliberately kept loose, resulting in slightly varied passing traces during multiple beam crossings.

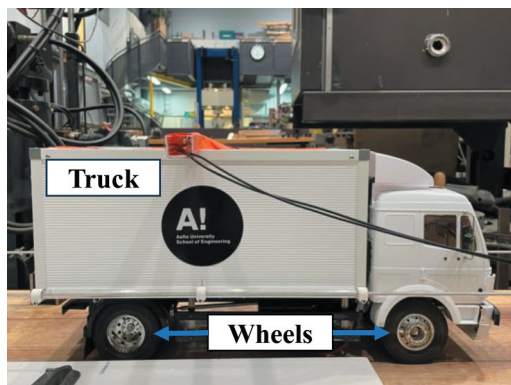


Figure 2. Model truck (lateral).

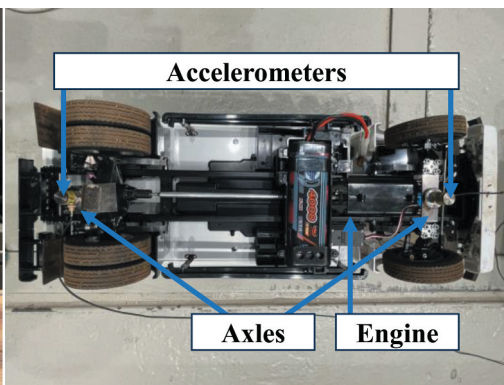


Figure 3. Model truck (bottom).

3.2 *Bridge model*

In this experiment, a single continuous beam with three supports is employed for the bridge configuration, as shown in Figure 4. The utilized beam is a UPE 300 with a cross-sectional area of 5660 mm², a span length of 5.7 m, a support length of 0.15 m, and a mass of 248.64 kg. To maintain a relatively constant speed as the truck traverses the beams, acceleration and deceleration runways are positioned at the beam ends. For comparison purposes, accelerometers are also installed at the bottom of the beam to capture its vibrations concurrently with the truck's passage. Additionally, to clearly identify the beam's modal frequencies, impulse excitation using an impact hammer is applied. Employing FFT analysis, the natural frequencies of the beam are obtained, as illustrated in Figure 5, revealing the first two frequencies as 30.75 and 42.53 Hz.

3.3 *Damage scenarios*

In practical engineering, the occurrence of damage can lead to a decrease in a bridge's natural frequencies. An effective experimental method for simulating damage involves adding mass to the bridge, a process known to cause a reduction in natural frequencies (Cerdeira et al., 2014).



Figure 4. Bridge model.

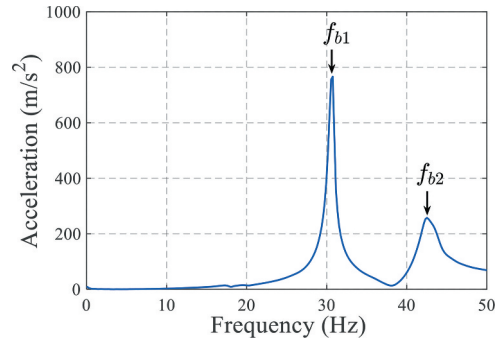


Figure 5. Bridge frequency spectrum.

This approach offers the advantage of easy recovery, allowing the beam to return to an undamaged state when necessary. In this study, various masses are applied to the bridge to simulate different artificial damage scenarios. Damage degrees are represented by the ratios of added masses to the beam’s mass. For example, when there are two 5 kg masses and two hooks (4 kg) added to the beam’s two spans, the damage degree is $(5+5+4)/248.64 = 5.6\%$. Table 1 summarizes all scenarios (S0-S6), where S0 means an undamaged bridge, while scenarios S1-S6 represent varying degrees of damage.

Table 1. Damage scenarios.

Scenarios	S0	S1	S2	S3	S4	S5	S6
Added mass/kg	0	5	10	15	20	25	30
Damage degree	0	5.6%	9.6%	13.7%	17.7%	21.7%	25.7%
T0 runs	50	49	50	42	51	51	47
T5 runs	57	56	57	56	57	56	57

4 RESULTS AND DISCUSSIONS

4.1 Model training and testing

In this study, runs of T0 or T5 are divided into two groups. 70% of runs are utilized for training and the rest are for testing. As outlined in Section 2, the vehicle’s time-domain signals on the bridge are transformed into the frequency domain. To mitigate contamination from environmental noises, only vehicle frequency-domain responses within 100 Hz are utilized for training, as validated in previous work (Li et al., 2023c). Zeros padding is applied to align input time-domain signals, resulting in an FFT frequency resolution of 0.0763 Hz, yielding 1310 response points within the 0-100 Hz range. The training is conducted on an Aalto University workstation equipped with Intel Core i7-9750 CPUs, 16 GB RAM, and an NVIDIA GTX 1650 graphic card for tensor computation acceleration. All code is implemented in Python 3.9 with the PyTorch package. The model architecture, detailed in Table 2, employs hyperparameters including a learning rate of $1e^{-4}$, Adam optimizer, cross-entropy loss, batch size of 16, and 200 epochs.

4.2 Damage classification using 1-D CNNs

When utilizing the vehicle’s frequency-domain responses within 0-100 Hz for training, loss curves for T0 and T5 vehicles have been plotted in Figures 6 and 7. Notably, for both T0 and T5 vehicles, employing responses from two axles yields smaller losses compared to using responses from only one axle. The difference is relatively minor for T0, as the lighter vehicle

Table 2. Architecture of the 1-D CNNs.

Layer	Pooling	Output shape	Batch normal	Activation
Conv1d	–	1310×20	Yes	ReLU
Conv1d	–	1310×20	Yes	ReLU
Conv1d	Max	655×20	Yes	ReLU
Conv1d	–	655×40	Yes	ReLU
Conv1d	–	655×40	Yes	ReLU
Conv1d	Max	327×40	Yes	ReLU
Conv1d	–	325×60	Yes	ReLU
Conv1d	–	325×40	Yes	ReLU
Conv1d	Avg	65×20	Yes	ReLU
Flattened	–	1300	No	–
Fully Connected	–	7	No	–

* Conv1d: 1-dimensional convolutional layer, ReLU: Rectified linear unit

may not induce strong vehicle-bridge interaction responses, resulting in less bridge information transferred to the vehicle. Despite including responses from another axle in the training, the model’s ability shows marginal improvement. In contrast, for the heavier T5 vehicle, the inclusion of responses from two axles significantly reduces the loss. This is attributed to the incorporation of valuable bridge damage information in the vehicle’s axles’ responses, enhancing the model’s damage classification capabilities.

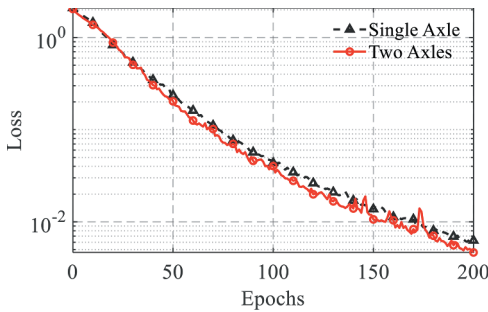


Figure 6. Loss curve using T0 vehicle.

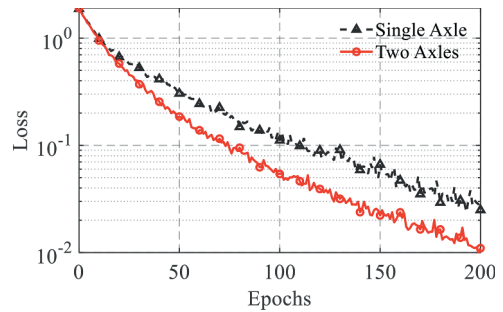


Figure 7. Loss curve using T5 vehicle.

To evaluate the capability of the trained model, some criteria listed in Eq. (4) are utilized,

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN}, Precision = \frac{TP}{TP + FP}, Recall = \frac{TP}{TP + FN} \quad (4)$$

where True Positive (TP) refers to a sample belonging to the positive class being classified correctly; True Negative (TN) refers to a sample belonging to the negative class being classified correctly; False Positive (FP) refers to a sample belonging to the negative class but being classified wrongly as belonging to the positive class; False Negative (FN) refers to a sample belonging to the positive class but being classified wrongly as belonging to the negative class. The damage classification accuracy using T0 and T5 are plotted in Figures 8 and 9.

We can see that for both T0 and T5 vehicles, employing responses from both axles yields higher accuracy compared to using responses from a single axle. Notably, with T0, accuracy stabilizes below 80% after 50 epochs. Conversely, Figure 9 illustrates that when a heavier vehicle is employed, accuracy consistently exceeds 95%. Consequently, in practical engineering

applications, it is advisable to recommend the use of heavier vehicles, as they can gather more damage-related information about the bridge.

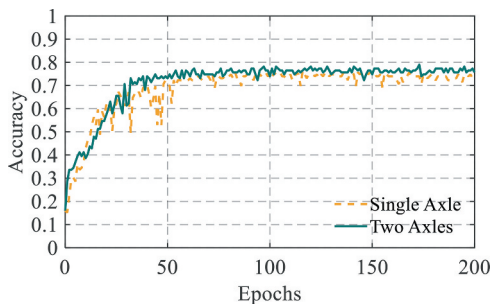


Figure 8. Damage classification accuracy (T0).

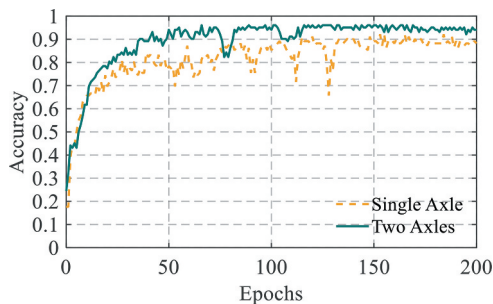


Figure 9. Damage classification accuracy (T5).

To assess the predictive capabilities of the trained DL model for damage severity classification, we employ the confusion matrix (CM), a widely used tool, to show the best bridge damage classification results. Figures 10 and 11 illustrate the CMs for T0 and T5, respectively.

From Figure 10, we can see that with the use of a light vehicle, the overall damage classification accuracy reaches 79%. Across most damage scenarios, **Recall** values range between 50% and 95%, though certain instances, such as S3, exhibit low **Recall**, indicating inaccuracies in predicting damage severity. Contrastingly, when employing a heavier vehicle, as shown in Figure 11, **Recall** values can consistently approach or exceed 90%, with some cases achieving 100% **Precision** and **Recall** values. The overall accuracy of damage classification across all scenarios increases significantly to 96.1%. Therefore, we can conclude that in practical engineering, heavier vehicles are recommended for the indirect bridge health monitoring using responses of passing vehicles.

4.3 Further discussions

In this study, we employ a scaled truck and a U-shaped beam to simulate real VBI scenarios. The road roughness, primarily caused by the wheels' tread, is relatively good compared to practical engineering cases. To alleviate the influence of road roughness, various techniques, such as CP responses and residual CP responses between wheels or connected vehicles, can be

True labels	S0	15 12.6%	1 0.8%	1 0.8%	0 0.0%	1 0.8%	0 0.0%	0 0.0%	83.3% 16.7%
	S1	0 0.0%	15 12.6%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	2 1.7%	88.2% 11.8%
	S2	0 0.0%	0 0.0%	12 10.1%	4 3.4%	1 0.8%	0 0.0%	0 0.0%	70.6% 29.4%
	S3	1 0.8%	0 0.0%	1 0.8%	9 7.6%	3 2.5%	1 0.8%	1 0.8%	56.2% 43.8%
	S4	0 0.0%	0 0.0%	0 0.0%	1 0.8%	14 11.8%	0 0.0%	0 0.0%	93.3% 6.7%
	S5	0 0.0%	0 0.0%	0 0.0%	4 3.4%	2 1.7%	15 12.6%	0 0.0%	71.4% 28.6%
	S6	0 0.0%	1 0.8%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	14 11.8%	93.3% 6.7%
Sum	93.8% 6.2%	88.2% 11.8%	85.7% 14.3%	50.0% 50.0%	66.7% 33.3%	93.8% 6.2%	82.4% 17.6%	79.0% 21.0%	
		S0	S1	S2	S3	S4	S5	S6	Sum
		Predicted labels							

Figure 10. Damage classification results (T0).

True labels	S0	14 13.7%	0 0.0%	1 1.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	93.3% 6.7%
	S1	0 0.0%	18 17.6%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	100% 0.0%
	S2	1 1.0%	0 0.0%	16 15.7%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	94.1% 5.9%
	S3	0 0.0%	0 0.0%	0 0.0%	13 12.7%	0 0.0%	0 0.0%	1 1.0%	92.9% 7.1%
	S4	0 0.0%	0 0.0%	0 0.0%	0 0.0%	17 16.7%	0 0.0%	0 0.0%	100% 0.0%
	S5	0 0.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	12 11.8%	0 0.0%	100% 0.0%
	S6	0 0.0%	1 1.0%	0 0.0%	0 0.0%	0 0.0%	0 0.0%	8 7.8%	88.9% 11.1%
Sum	93.3% 6.7%	94.7% 5.3%	94.1% 5.9%	100% 0.0%	100% 0.0%	100% 0.0%	88.9% 11.1%	96.1% 3.9%	
		S0	S1	S2	S3	S4	S5	S6	Sum
		Predicted labels							

Figure 11. Damage classification results (T5).

examined. Furthermore, it is noted that as an initial investigation, this paper mainly explored the superior capability when responses of multiple positions of the vehicle are utilized. To overcome the challenges that labeled damaged data are difficult to obtain in engineering, semi-supervised or unsupervised DL techniques deserve further studies.

5 CONCLUSIONS

This paper proposed a bridge damage classification strategy using responses of the passing vehicle's multiple positions and 1-D CNNs. Firstly, the fundamental theories about CNNs are introduced. Then, the proposed method was verified via a scaled truck and continuous beam in laboratory experiments. The experimental results indicate that the bridge's damage severity can be predicted by the CNN with high accuracy, and some concluding remarks are drawn below:

- (1) The passing vehicle's responses on multiple positions can contain bridge damage information. Therefore, in the DL model training process, the responses from more DOFs are supposed to be included to increase its damage classification capability.
- (2) Heavier vehicles can make the VBI interaction responses stronger. Through this, more bridge damage-related information can be transferred to the passing vehicles' vibrations, resulting in boosted damage classification accuracy.

Even though there are some key findings, some important influence factors, such as ongoing traffic and very poor road roughness, have not been checked. Our future studies will include these factors and further unsupervised DL models before engineering applications.

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